Table 1
some reduced properties of classical and " quantized " ljd liquids at gero pressure

| $\begin{aligned} & \text { LJD } \\ & \text { Liquids } \end{aligned}$ | Temperature $T^{*}$ | $\begin{gathered} \text { Volume } \\ V^{*} \end{gathered}$ | Coefficient of Thermal Expansion $\frac{1}{V^{*}}\left(\frac{\partial V^{*}}{\partial T^{*}}\right)_{P}$ | Isothermal Compressibility $-\frac{1}{V^{*}}\left(\frac{\partial V^{*}}{\partial P^{*}}\right)_{T}$ | Heat Capacity $C_{V}^{*}$ | Speed of Sound $u^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) Classical: $\Lambda^{*}=0$ | $\begin{aligned} & 0 \\ & 0 \cdot 70 \\ & 0 \cdot 75 \\ & 0 \cdot 80 \\ & 0 \cdot 85 \\ & 0 \cdot 90 \\ & 0 \cdot 95 \\ & 1 \cdot 00 \end{aligned}$ | $\begin{aligned} & 0 \cdot 916 \\ & 1 \cdot 037 \\ & 1 \cdot 050 \\ & 1 \cdot 065 \\ & 1 \cdot 081 \\ & 1 \cdot 099 \\ & 1 \cdot 120 \\ & 1 \cdot 145 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \cdot 244 \\ & 0 \cdot 261 \\ & 0 \cdot 287 \\ & 0 \cdot 316 \\ & 0 \cdot 352 \\ & 0 \cdot 405 \\ & 0 \cdot 491 \end{aligned}$ | $\begin{aligned} & 0.0133 \\ & 0.0348 \\ & 0.0386 \\ & 0.0433 \\ & 0.0493 \\ & 0.0571 \\ & 0.0683 \\ & 0.0851 \end{aligned}$ | $\begin{aligned} & 0 \\ & 2 \cdot 61 \\ & 2 \cdot 58 \\ & 2 \cdot 55 \\ & 2 \cdot 53 \\ & 2 \cdot 50 \\ & 2 \cdot 47 \\ & 2 \cdot 43 \end{aligned}$ | $\begin{aligned} & 8 \cdot 30 \dagger \\ & 6 \cdot 63 \\ & 6 \cdot 47 \\ & 6 \cdot 34 \\ & 6 \cdot 17 \\ & 5 \cdot 98 \\ & 5 \cdot 78 \\ & 5 \cdot 60 \end{aligned}$ |
| (b) Quantal: $\Lambda^{*}=0.5$ | $\begin{aligned} & 0.70 \\ & 0.75 \\ & 0.80 \\ & 0.85 \\ & 0.90 \\ & 0.95 \\ & 1.00 \end{aligned}$ | $\begin{aligned} & 1 \cdot 090 \\ & 1 \cdot 109 \\ & 1 \cdot 130 \\ & 1 \cdot 155 \\ & 1 \cdot 186 \\ & 1 \cdot 226 \\ & 1 \cdot 290 \end{aligned}$ | $\begin{aligned} & 0 \cdot 319 \\ & 0 \cdot 358 \\ & 0 \cdot 408 \\ & 0 \cdot 477 \\ & 0 \cdot 584 \\ & 0 \cdot 784 \\ & 1 \cdot 366 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0454 \\ & 0 \cdot 0527 \\ & 0 \cdot 0626 \\ & 0 \cdot 0771 \\ & 0 \cdot 1004 \\ & 0 \cdot 1458 \\ & 0 \cdot 286 \end{aligned}$ | $\begin{aligned} & 2 \cdot 71 \\ & 2 \cdot 68 \\ & 2 \cdot 64 \\ & 2 \cdot 59 \\ & 2 \cdot 54 \\ & 2 \cdot 47 \\ & 2 \cdot 38 \end{aligned}$ | $\begin{aligned} & 6 \cdot 00 \\ & 5 \cdot 77 \\ & 5 \cdot 52 \\ & 5 \cdot 24 \\ & 4 \cdot 92 \\ & 4 \cdot 53 \\ & 3 \cdot 99 \end{aligned}$ |
| (c) Quantal: $\Lambda^{*}=1 \cdot 0$ | $\begin{aligned} & 0.70 \\ & 0.75 \\ & 0.80 \\ & 0.85 \end{aligned}$ | $\begin{aligned} & 1 \cdot 213 \\ & 1 \cdot 245 \\ & 1 \cdot 288 \\ & 1 \cdot 353 \end{aligned}$ | $\begin{aligned} & 0 \cdot 474 \\ & 0 \cdot 586 \\ & 0 \cdot 780 \\ & 1 \cdot 287 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0819 \\ & 0 \cdot 1061 \\ & 0 \cdot 1516 \\ & 0 \cdot 280 \end{aligned}$ | $\begin{aligned} & 2 \cdot 46 \\ & 2 \cdot 47 \\ & 2 \cdot 45 \\ & 2 \cdot 38 \end{aligned}$ | $\begin{aligned} & 4 \cdot 87 \\ & 4 \cdot 58 \\ & 4 \cdot 22 \\ & 3 \cdot 71 \end{aligned}$ |

$\dagger$ This value was derived previously (Hamann 1960).
Our calculations on quantal liquids were limited to values of $\Lambda^{*}$ less than 1.5 for two reasons:
(i) If $\Lambda^{*}$ is much greater than one, the stable range of the liquid state is shifted to lower reduced temperatures than are covered by the tables of Wentorf et al. (1950).
(ii) Equations (12) and (15) were based on an Euler-Maclaurin expansion of the partition function (Hamann 1952) which is only valid when $x^{*}$ is greater than $1 \cdot 5$. If $\Lambda^{*}$ is large, $x^{*}$ becomes less than this value. We have therefore not been able to apply the theory directly to $\mathrm{H}_{2}$ and the helium isotopes, but the trend of the curves to $\Lambda^{*}=1$ is certainly sufficient to explain the behaviour of the lighter liquids.

## (b) Liquids at High Pressures

The computation of $u^{*}$ is easily extended to compressed liquids. Using the polynomial form (10) of the $P^{*}-V^{*}$ relation, we can derive values of the derivative $\left(\partial P^{*} / \partial V^{*}\right)_{T}$ over a wide range of temperatures and densities. As before, the derivative $\left(\partial P^{*} / \partial T^{*}\right)_{V}$ and the corresponding values of $P^{*}$ and $C_{V}^{*}$ can be taken directly from the tables of Wentorf et al. (1950). The results are


Fig. 1.-A comparison of the calculated and experimental speeds of sound in simple liquids. The sources of the experimental data for A, $\mathrm{N}_{2}, \mathrm{O}_{2}, \mathrm{CH}_{4}, \mathrm{H}_{2}$, and $\mathrm{He}^{4}$ have been given in an earlier paper (Hamann 1960). The data for $\mathrm{He}^{3}$ have been taken from a paper by Atkins and Flicker (1959).


Fig. 2.-The effect of pressure on the speed of sound. The curves represent the theoretical Leonard-Jones-Devonshire relations and the crosses denote the experimental data for argon at $T^{*}=0 \cdot 75$.

